Stochastic resonance in two-state model of membrane channel with comparable opening and closing rates

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Voltage-gated ion channels in biological membranes can be modeled as two-state stochastic systems with inter-state transition probabilities that depend on external stimulus. We study analytically the passage of a signal through such a system in the presence of external noise in the case when the above dependence is arbitrary, and illustrate our approach using the two models of ion channels known in the literature. The explicit expressions for the spectral density of the output signal and noise, the signal-to-noise ratio, and the coherence function are obtained for rectangular periodic signal and dichotomous noise in a wide range of parameters. The dependence of the above quantities on the bias and on the noise amplitude demonstrates strong resonant behavior in the regions where the probabilities of channel closing and opening become equal. This resonance results from additional symmetry between channel states and differs from conventional stochastic resonance studied earlier.

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I. INTRODUCTION

Stochastic resonance (SR) is one of the most interesting and bright examples of the constructive role of noise in nature. This phenomenon has been extensively studied in the last decade (see reviews [1,2] and the older review [3]) and can be briefly described as the optimization of passage of a signal through a nonlinear system with the noise of nonzero intensity, added to the signal. SR is found out in numerous systems of various kinds; a significant fraction of publications was devoted to the study of its possible role in information transfer in living organisms, where the useful signals are always accompanied by strong ineradicable noise (see the recent minireview [4]). In particular, it was found out that living beings can use SR at several levels: the system one (the recognition of visual [5] and tactile [6] stimuli by humans) and the cellular one (mechanoreceptors of crayfish [7], neurons of brain cortex [8], ear hair cells [9,10]). Bezrukov and Vodyanoy in their experiment [11] studied the ensemble of ion channels, each of them being a small aggregate of alamethicin molecules, in a bilayer lipid membrane. They discovered that the system amplifies a small low-frequency periodic electric signal by several orders of magnitude after addition of high-frequency colored noise to the signal. They also observed a weak maximum of the signal-to-noise ratio at an optimum noise intensity. Thus the possibility of using SR at a molecular level in biological systems was demonstrated (the first, unsuccessful attempt to find SR in a single ion channel was performed in [12]). In further publications the authors showed analytically that SR could be observed in practically any system that generates stochastic pulses with nonlinear (exponential, in particular) dependence of the rate of pulse appearance on external stimulus [13,14]. The theory is constructed for a case of rather rare pulses of negligibly small duration. In case of a single ion channel, the time course of its conductivity can be roughly represented as the sequence of random switchings between two states, the open one and the closed one. The situation described by the above theory is realized here when the probability of transition from one channel state to another (i.e., closing) is much higher than the probability of backwards transition (opening). In this case the channel is closed most of the time, opening only occasionally and briefly. Such a set of properties is not typical for channels in membranes of living cells. In many cases, the opening and closing rates and corresponding dwell times are comparable [15]. The last circumstance is noted by Goychuk and Hänggi in [16] where they studied SR by information theory methods in the model of ion channel, considering it as a two-state system with an arbitrary ratio of transition rates. The theory is restricted by the case of an adiabatically slow signal and white noise. Our work is devoted to the analytical study of passage of a signal through an ion channel as the two-state system where the transition probabilities between states depend on external stimulus. The theory is constructed for arbitrary form of such dependence; for illustrative purposes we choose the model of potassium channel used in [16] [see Eq. (2) below], as well as the widely used model of Hodgkin-Huxley type [15] [our Eq. (33)].We study the system with dichotomous noise and rectangular periodic signal. Such a choice of noise and signal allows us to solve the problem exactly for any, not necessarily small, values of noise and signal amplitudes. We obtain explicit expressions for the signal-to-noise ratio (SNR), the coherence function Γ^2, and the power of output signal and noise in a wide range of parameters. The main result of the paper is the detection of resonant behavior of investigated quantities in the case when the probabilities of opening and closing of the channel become equal. This resonance differs from conventional stochastic resonance; it is related to an additional symmetry between open and closed states of the channel that appears when the transition probabilities are identical. In the range of existence of such symmetry all the characteristics display interesting nonmonotonous behavior.

II. THE MODEL

The ion channel can be described by a simple dichotomous model [15]. Let $g(t)$ be the time-dependent conductiv-

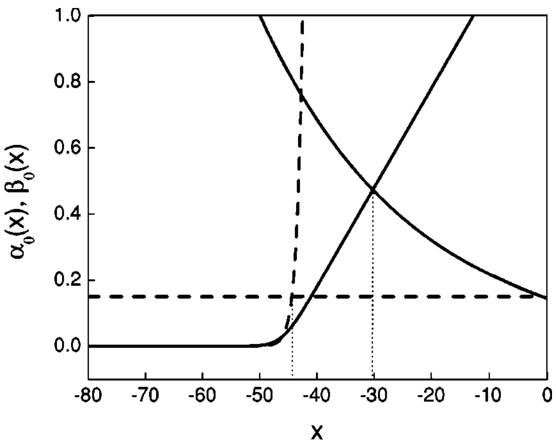

FIG. 1. The transition probabilities $\alpha_0(x)$, $\beta_0(x)$ for the model of potassium channel (2) (solid curves) and for the model of Hodgkin-Huxley type (33) (dashed curves) for the values of parameters: $p_2=46$, $a_1=0.15$, $a_2=0.03$, $b_1=0.038$, and $b_2=1$.

ity of the channel. In the standard theory it can accept two values, 0 and g_0. For our purposes it is convenient to introduce the dichotomous variable $d(t)$:

$$d(t)=\pm 1, \quad g(t)=\frac{g_0}{2}\{1+d(t)\}. \tag{1}$$

Obviously, $d(t)=-1$ corresponds to the closed state of the channel, and $d(t)=1$ to the open one. The transitions between these states are determined by transition probabilities, which, in turn, adiabatically follow the external stimulus (the transmembrane voltage drop) $x(t)$. Let us define $\alpha(t)$ as the probability of transition of the channel from the closed state to the open one, and $\beta(t)$ as the probability of backwards transition. The dependence of these quantities on voltage has the form similar to that used by Goychuk and Hänggi [16]:

$$\alpha(t)=\alpha_0\{x(t)\}, \quad \beta(t)=\beta_0\{x(t)\},$$
$$\alpha_0(x)=\frac{a_2(x+p_2)}{1-\exp\{-b_2(x+p_2)\}}, \quad \beta_0(x)=a_1\exp(-b_1 x). \tag{2}$$

The dependences (2) are represented in Fig. 1.

The external stimulus $x(t)$ consists of the bias p, the dichotomous noise $s(t)$, and the square-wave periodic signal $R(t)$:

$$x(t)=p+us(t)+vR(t),$$
$$s(t)=\pm 1, \quad \langle s(t)\rangle=0,$$
$$\langle s(t)\, s(t')\rangle=\exp\{-\gamma|t-t'|\}, \tag{3}$$
$$R(t+T)=R(t)=\begin{cases}1, & 0<t\leqslant T/2,\\ -1, & T/2<t\leqslant T.\end{cases}$$

In the present work we scale voltages and times to their characteristic values 1 mV and 1 ms, like the authors of [16].

Now let us note that there are two kinds of stochasticity in our problem. The internal one is related to transitions between two states $d(t)$, and the external one is caused by external noise $s(t)$. As a result, two types of averaging appear in the theory, which we further designate, accordingly, $\langle\,\rangle_d$ and $\langle\,\rangle_s$.

Besides, there are three characteristic times in the problem: the switching time of $d(t)$, equal to a_i^{-1}, the time of correlation of dichotomous noise γ^{-1}, and the period of signal T. We take $T\gg\gamma^{-1},a_i^{-1}$, however, allowing the ratio between γ and a_i to vary. We shall consider two cases, the adiabatic one:

$$T\gg\gamma^{-1}\gg a_i^{-1}, \tag{4}$$

and the fast noise one:

$$T\gg a_i^{-1}\gg\gamma^{-1}. \tag{5}$$

In both these limits it is possible to explicitly calculate all quantities that we are interested in as functions of parameters of the stimulus p,u,v.

III. ADIABATIC CASE

The master equation for nonstationary probability density $P(d,t)$ for our problem is

$$\frac{dP(1,t)}{dt}=\alpha(t)P(-1,t)-\beta(t)P(1,t),$$
$$\frac{dP(-1,t)}{dt}=\beta(t)P(1,t)-\alpha(t)P(-1,t). \tag{6}$$

Since the local balance in the system in adiabatic conditions is established much faster than the transition probabilities change, the solution of Eq. (6) reads as

$$P(1,t)=\frac{\alpha(t)}{\alpha(t)+\beta(t)}, \quad P(-1,t)=\frac{\beta(t)}{\alpha(t)+\beta(t)}. \tag{7}$$

The probability of transition from state m at the moment t_2 to state n at the moment $t_1>t_2$ can be also easily calculated:

$$P(n,t_1|m,t_2)=P(n,t_1)+\frac{\varphi(n,t_1)\varphi(m,t_1)}{P(m,t_1)}\times\exp\{-(t_1-t_2)\lambda_0(t_1)\},$$
$$\lambda_0(t)=\alpha(t)+\beta(t), \tag{8}$$
$$\varphi(1,t)=-\varphi(-1,t)=\frac{\sqrt{\alpha(t)\beta(t)}}{\alpha(t)+\beta(t)}.$$

In Eq. (8) $\lambda_0(t)$ and $\varphi(n,t)$ are the adiabatically slowly varying nonzero eigenvalue and the corresponding eigenfunction of Eq. (6). Let us calculate now the two-particle probability density function (PDF) $P_2(n,t_1|m,t_2)$. According to the common theory of Markov processes, this PDF is

$$P_2(n,t_1|m,t_2)=P(n,t_1|m,t_2)P(m,t_2)=P(n,t_1)P(m,t_2)+\frac{\varphi(n,t_1)\varphi(m,t_1)}{P(m,t_1)}P(m,t_2)\times\exp\{-(t_1-t_2)\lambda_0(t_1)\}. \tag{9}$$

The time range of our interest is

$$|t_1-t_2|\sim T, \quad \gamma^{-1}\gg a_i^{-1}. \tag{10}$$

For these times we can replace the exponent in Eq. (9) by a delta function and therefore obtain the following expression:

$$P_2(n,t_1|m,t_2)=P(n,t_1)P(m,t_2)+\frac{2}{\lambda_0(t_1)}\varphi(n,t_1)\varphi(m,t_1)\delta(t_1-t_2). \tag{11}$$

Now, taking into account Eqs. (7) and (8), we get the autocorrelation function

$$D(t_1,t_2)=\langle d(t_1)d(t_2)\rangle_d=y_0(t_1)y_0(t_2)+\frac{8\alpha(t_1)\beta(t_1)}{[\alpha(t_1)+\beta(t_1)]^3}\delta(t_1-t_2),$$

$$y_0(t)=\langle d(t)\rangle_d=\frac{\alpha(t_1)-\beta(t_1)}{\alpha(t_1)+\beta(t_1)}\equiv y_0[x(t)], \tag{12}$$

$$y_0(x)=\frac{\alpha_0(x)-\beta_0(x)}{\alpha_0(x)+\beta_0(x)}.$$

Let us now average Eq. (12) by stochastic process $s(t)$ and by phase of periodic process $R(t)$. We calculate the following autocorrelator:

$$K(t_1-t_2)=\langle D(t_1,t_2)\rangle_{s,R}-\langle y_0(t_1)\rangle_{s,R}\langle y_0(t_2)\rangle_{s,R}. \tag{13}$$

From Eqs. (12) and (13) we obtain

$$K(t_1-t_2)=\langle d(t_1)\,d(t_2)\rangle_{d,s,R}-\langle d(t_1)\rangle_{d,s,R}\langle d(t_2)\rangle_{d,s,R}=\langle y_0(t_1)y_0(t_2)\rangle_{s,R}-\langle y_0(t_1)\rangle_{s,R}\langle y_0(t_2)\rangle_{s,R}+8\left\langle\frac{\alpha(t_1)\beta(t_1)}{[\alpha(t_1)+\beta(t_1)]^3}\right\rangle_{s,R}\delta(t_1-t_2). \tag{14}$$

It is seen from Eqs. (1) and (14) that

$$G(t_1-t_2)=\langle g(t_1)\,g(t_2)\rangle_{d,s,R}-\langle g(t_1)\rangle_{d,s,R}\langle g(t_2)\rangle_{d,s,R}=\frac{g_0^2}{4}K(t_1-t_2)=\langle y(t_1)y(t_2)\rangle_{s,R}-\langle y(t_1)\rangle_{s,R}\langle y(t_2)\rangle_{s,R}+2g_0^2\left\langle\frac{\alpha(t_1)\beta(t_1)}{[\alpha(t_1)+\beta(t_1)]^3}\right\rangle_{s,R}\delta(t_1-t_2),$$

$$y(t)=\langle g(t)\rangle_d=\frac{g_0}{2}[1+y_0(t)]=g_0\frac{\alpha(t)}{\alpha(t)+\beta(t)}. \tag{15}$$

From Eq. (7) one can see that, first, $y(t)/g_0$ is the probability that the channel is open, and, second, the autocorrelation function of conductivity is simply related to the one of dichotomous variable. We study the behavior of two characteristics of the system. In the presence of a periodic signal we can use the signal-to-noise ratio. Otherwise, it is convenient to use the coherence function

$$\Gamma^2(\omega)=\frac{|S_1(\omega)|^2}{S(\omega)S_2(\omega)},$$

$$S(\omega)=\int e^{i\omega t}K(t)dt, \quad S_j(\omega)=\int e^{i\omega t}K_j(t)dt,$$

$$j=1,2, \tag{16}$$

$$K_1(t_1-t_2)=\langle d(t_1)S(t_2)\rangle_{d,s,R}=\langle y_0(t_1)S(t_2)\rangle_{s,R},$$

$$K_2(t_1-t_2)=\langle S(t_1)S(t_2)\rangle_S=\exp[-\gamma|t_1-t_2|].$$

In the presence of both the dichotomous noise and rectangular signal it is possible to obtain expressions for all quantities of our interest. At first, we calculate the autocorrelation function $K(t)$. Since both signal and noise take only two values ±1, we get, taking into account Eq. (3), for arbitrary function $f\{x(t)\}$:

$$f\{x(t)\}=\frac{1+s(t)}{2}\frac{1+R(t)}{2}f(p+u+v)+\frac{1-s(t)}{2}\frac{1-R(t)}{2}f(p-u-v)+\frac{1+s(t)}{2}\frac{1-R(t)}{2}f(p+u-v)+\frac{1-s(t)}{2}\frac{1+R(t)}{2}f(p-u+v)$$
$$=f_0(p,u,v)+f_1(p,u,v)s(t)+f_2(p,u,v)R(t)+f_3(p,u,v)s(t)R(t),$$

$$f_0(p,u,v)=\tfrac{1}{4}\{f(p+u+v)+f(p-u-v)+f(p+u-v)+f(p-u+v)\}, \tag{17}$$

$$f_1(p,u,v)=\tfrac{1}{4}\{f(p+u+v)-f(p-u-v)+f(p+u-v)-f(p-u+v)\},$$

$$f_2(p,u,v)=\tfrac{1}{4}\{f(p+u+v)-f(p-u-v)-f(p+u-v)+f(p-u+v)\},$$

$$f_3(p,u,v)=\frac{1}{4}\{f(p+u+v)+f(p-u-v)-f(p+u-v)-f(p-u+v)\}.$$

The above coefficients have the following symmetry properties:

$$f_1(p,0,v)=0,\quad f_2(p,u,0)=0,$$
$$f_3(p,0,v)=f_3(p,u,0)=0. \tag{18}$$

We can now easily get the following expression for arbitrary functions $f^{(1)},f^{(2)}$ of the random process $x(t)$:

$$\langle f^{(1)}\{x(t)\}f^{(2)}\{x(0)\}\rangle_{s,R}-\langle f^{(1)}\{x(t)\}\rangle_{s,R}\langle f^{(2)}\{x(0)\}\rangle_{s,R}$$
$$=f_1^{(1)}(p,u,v)f_1^{(2)}(p,u,v)e^{-\gamma|t|}+f_2^{(1)}(p,u,v)f_2^{(2)}(p,u,v)\varphi_0(t)+f_3^{(1)}(p,u,v)f_3^{(2)}(p,u,v)e^{-\gamma|t|}\varphi_0(t), \tag{19}$$

$$\varphi_0(t)=\langle R(t)R(0)\rangle_R=\frac{4}{\pi^2}\sum_{k=0}^{\infty}(2k+1)^{-2}\times\exp[-i(2k+1)\Omega t],\quad \Omega=\frac{2\pi}{T}.$$

In our calculations of correlation functions we should obviously take $f^{(1)}=f^{(2)}=y_0(t)$ for $K(t)$, and $f^{(1)}=y_0(t),f^{(2)}=s(t)$ for $K_1(t)$. We also take $v=0$ when calculating $K_1(t)$. Thus we obtain, after some calculations,

$$K(t)=B_1(p,u,v)e^{-\gamma|t|}+B_2(p,u,v)\varphi_0(t)+B_3(p,u,v)e^{-\gamma|t|}\varphi_0(t)+C(p,u,v)\delta(t),$$
$$K_1(t)=\sqrt{B_1(p,u,0)}e^{-\gamma|t|},$$
$$B_1(p,u,v)=\frac{1}{16}\{y_0(p+u+v)-y_0(p-u-v)+y_0(p+u-v)-y_0(p-u+v)\}^2,$$
$$B_2(p,u,v)=\frac{1}{16}\{y_0(p+u+v)-y_0(p-u-v)-y_0(p+u-v)+y_0(p-u+v)\}^2, \tag{20}$$
$$B_3(p,u,v)=\frac{1}{16}\{y_0(p+u+v)+y_0(p-u-v)-y_0(p+u-v)-y_0(p-u+v)\}^2,$$
$$C(p,u,v)=\langle C_0(x)\rangle_{s,R}=\frac{1}{4}\{C_0(p+u+v)+C_0(p-u-v)+C_0(p+u-v)+C_0(p-u+v)\},$$
$$C_0(x)=\frac{8\alpha_0(x)\beta_0(x)}{[\alpha_0(x)+\beta_0(x)]^3}.$$

Equation (20) gives the complete solution of the problem in the range of parameters $T,\gamma^{-1}\gg a_i^{-1}$. However, we are interested only with the adiabatic range Eq. (4), where we can assume

$$\varphi_0(t)e^{-\gamma|t|}\approx\varphi_0(0)e^{-\gamma|t|}=e^{-\gamma|t|}. \tag{21}$$

Thus we get from Eq. (19)

$$K(t)=\{B_1(p,u,v)+B_3(p,u,v)\}e^{-\gamma|t|}+B_2(p,u,v)\varphi_0(t)+C(p,u,v)\delta(t). \tag{22}$$

The noise background in the adiabatic case can be calculated simply at $\omega=0$, since $\Omega\ll\gamma$. Therefore we obtain

$$S(\omega)=S(0)+B_2(p,u,v)\varphi_0(\omega),$$
$$S(0)=\frac{2}{\gamma}\{B_1(p,u,v)+B_3(p,u,v)\}+C(p,u,v), \tag{23}$$
$$\varphi_0(\omega)=\frac{8}{\pi}\sum_{k=0}^{\infty}(2k+1)^{-2}\delta[\omega-(2k+1)\Omega].$$

We define the signal-to-noise ratio as the ratio of the power of fundamental harmonics of the signal to the noise background:

$$\mathcal{R}_{\mathrm{SN}}=\frac{8}{\pi}\frac{B_2(p,u,v)}{C(p,u,v)+\frac{2}{\gamma}[B_1(p,u,v)+B_3(p,u,v)]}. \tag{24}$$

The maximum value of coherence function (at zero frequency) is

$$\Gamma^2(0)=\frac{B_1(p,u,0)}{B_1(p,u,0)+\frac{\gamma}{2}C(p,u,0)}. \tag{25}$$

From Eqs. (24) and (25) we see that the term with C is small by the parameter of adiabaticity γa_i^{-1}. Nevertheless, we take it into account, since one can see from Eq. (18) that in the range of small noise amplitudes the coefficients B_1,B_3 are small, and the contribution from the considered term dominates. So, the formulas (20), (23)–(25) determine the characteristics of the system in the adiabatic case.

IV. FAST NOISE CASE

When the external noise is fast [Eq. (5)],it is known [19] that the time-dependent rates $\alpha(t)$ and $\beta(t)$ in Eq. (6) can be treated as nonrandom variables. This means that we can substitute their instantaneous values in Eq. (6) with noise-averaged ones. Therefore, we get the following master equations:

$$\frac{dP(1,t)}{dt}=\alpha_1(t)P(-1,t)-\beta_1(t)P(1,t),$$
$$\frac{dP(-1,t)}{dt}=\beta_1(t)P(1,t)-\alpha_1(t)P(-1,t),$$
$$\alpha_1(t)=\langle\alpha(t)\rangle_S=\alpha_2[x_1(t),u],$$

$$\beta_1(t)=\langle\beta(t)\rangle_S=\beta_2[x_1(t),u], \tag{26}$$

$$\alpha_2[x_1,u]=\tfrac{1}{2}[\alpha_0(x_1+u)+\alpha_0(x_1-u)],$$

$$\beta_2[x_1,u]=\tfrac{1}{2}[\beta_0(x_1+u)+\beta_0(x_1-u)],$$

$$x_1(t)=p+vR(t).$$

The adiabatic condition for the external signal $vR(t)$ is still valid. Since Eqs. (26) do not contain $s(t)$, we should repeat the calculations of the preceding section only for $R(t)$. The terms with B_1, B_3 now disappear, and we get

$$K_1(t)\to 0, \quad \Gamma^2(\omega)\to 0. \tag{27}$$

The expression for the autocorrelation function of the dichotomous variable becomes simpler:

$$K(t)=B(p,u,v)\varphi_0(t)+C_1(p,u,v)\delta(t),$$

$$B(p,u,v)=\tfrac{1}{4}[y_2(p+v,u)-y_2(p-v,u)]^2,$$

$$y_2(x_1,u)=\frac{\alpha_2(x_1,u)-\beta_2(x_1,u)}{\alpha_2(x_1,u)+\beta_2(x_1,u)}, \tag{28}$$

$$C_1(p,u,v)=\tfrac{1}{2}[C_2(p+v,u)+C_2(p-v,u)],$$

$$C_2(x_1,u)=\frac{8\alpha_2(x_1,u)\beta_2(x_1,u)}{[\alpha_2(x_1,u)+\beta_2(x_1,u)]^3}.$$

The signal-to-noise ratio is

$$\mathcal{R}_{\rm SN}=\frac{8}{\pi}\frac{B}{C_1}. \tag{29}$$

We note that Eq. (28) differs from Eq. (20) by the order of averaging. In Eq. (28) the transition probabilities are averaged by fast noise first, and subsequently the final expressions are averaged by the phase of the periodic signal. In Eq. (20), however, only the final expressions are averaged simultaneously by the noise and the phase of signal.

V. ANALYSIS OF THE RESULTS

First of all, we show that the behavior of the noise background $S(0)$ and the output signal power B_2 in the adiabatic case is rather universal and almost independent of the particular form of transition probabilities dependence on external stimulus. Let us consider the case of a small signal:

$$v\ll 1, \quad p,u\gg 1. \tag{30}$$

Now we see from Eq. (23) that, since $B_3\ll B_1$ and $B_1\gg\gamma C$ at $\gamma\ll a_i$,

$$S(0)=\frac{2}{\gamma}B_1(p,u,0),$$

$$B_1(p,u,0)=\tfrac{1}{4}[y_0(p+u)-y_0(p-u)]^2, \tag{31}$$

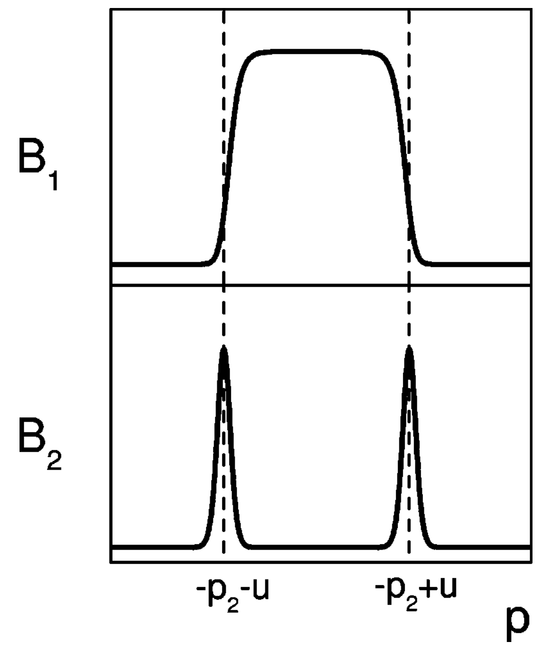

FIG. 2. The sketch of B_1, B_2 vs the bias p.

$$B_2(p,u,v\ll 1)=\frac{v^2}{4}[y_0'(p+u)+y_0'(p-u)]^2,$$

$$\mathcal{R}_{\rm SN}=\frac{4\gamma}{\pi}\frac{B_2}{B_1}.$$

It can be seen from the definition of $y_0(x)$ in Eq. (12) that

$$y_0(x)=\begin{cases} 1-\dfrac{2\beta_0(x)}{\alpha_0(x)}, & \beta_0(x)\ll\alpha_0(x),\\[2ex] -1+\dfrac{2\alpha_0(x)}{\beta_0(x)}, & \beta_0(x)\gg\alpha_0(x),\end{cases} \tag{32}$$

that is, this function changes from -1 to 1 in the region where the curves $\alpha_0(x)$ and $\beta_0(x)$ intersect. One can see from Fig. 1 that the intersection point locates near $x\approx -p_2$. Thus the rapid change of $y_0(x)$ and the peak of $y_0'(x)$ occur for this value of x. The sketch in Fig. 2 shows that the behavior of B_1, B_2 in Eq. (31) in the vicinity of the points $p+p_2\pm u=0$ is strongly nonmonotonous. This behavior is also rather universal, since the locations of the mentioned points are determined by intersection of $\alpha_0(x)$ and $\beta_0(x)$ and depend only weakly on the particular form of the latter curves. Remember that the described universality is observed for small and adiabatically slow signals. Figure 3 demonstrates the dependences of the output signal power, the noise background, and the signal-to-noise ratio on the bias p and the external noise amplitude u. For large p and u the two first characteristics show the same behavior as in Fig. 2. Figure 4 displays the sections of the SNR surface for various bias values. It is clearly seen that the maximum of the SNR is observed at $p+p_2\pm u=0$. We see also from Figs. 3 and 4 that for small noise amplitude the best conditions for signal transmission are achieved at $p+p_2\approx 0$. To emphasize the

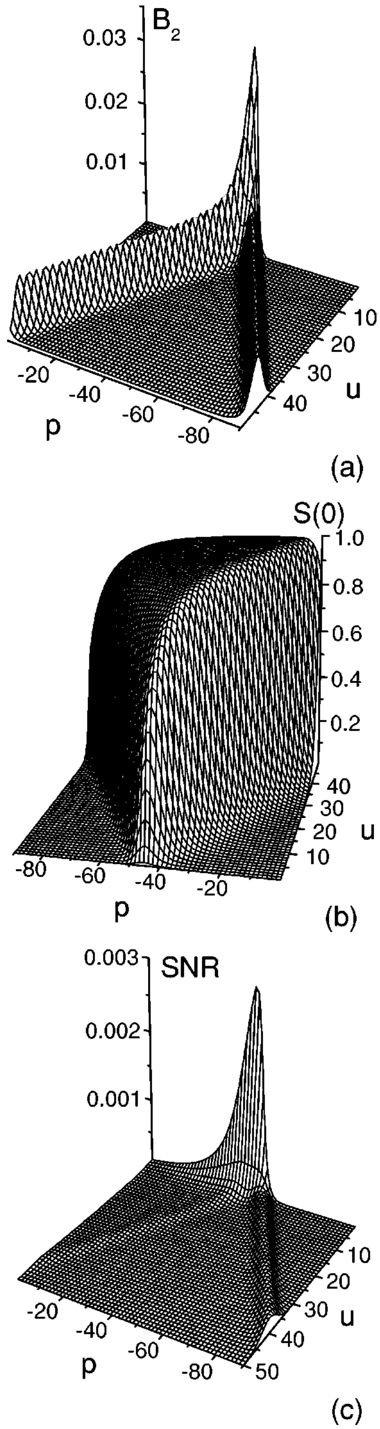

FIG. 3. The dependence of output signal (a), the noise background (b), and SNR (c) on the bias p and the noise amplitude u in the adiabatic case [Eqs. (20), (23), (24)] for the model (2) at $\gamma = 0.001$, $p_2 = 46$, $a_2 = 0.03$, $b_2 = 1$, $a_1 = 0.015$, $b_1 = 0.038$, and $v = 1$. In accordance with [16], the values of voltages p,u, and v are scaled to 1 mV, and those of times to 1 ms. (b) is turned in the plane (p,u) with respect to the other two; the noise-induced noise amplification is evident.

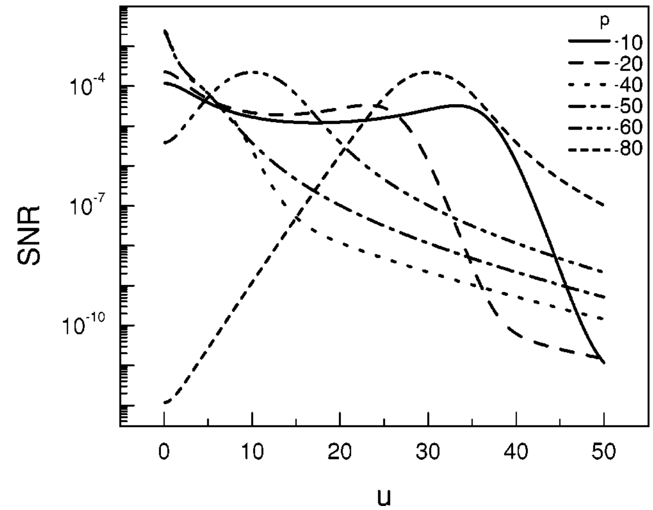

FIG. 4. The signal-to-noise ratio vs the noise amplitude u for the model (2) in the adiabatic case for several values of the bias p. The values of parameters are the same as in Fig. 3.

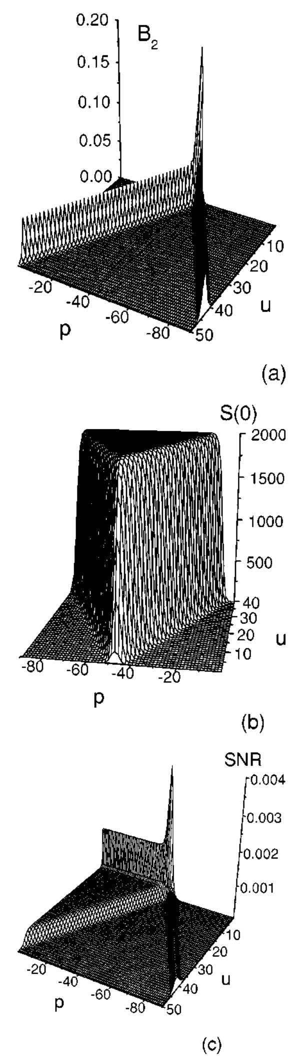

FIG. 5. The same dependences as in Fig. 3 but for the model (33). The values of parameters a_2, p_2, a_1, b_2, and γ are the same as above.

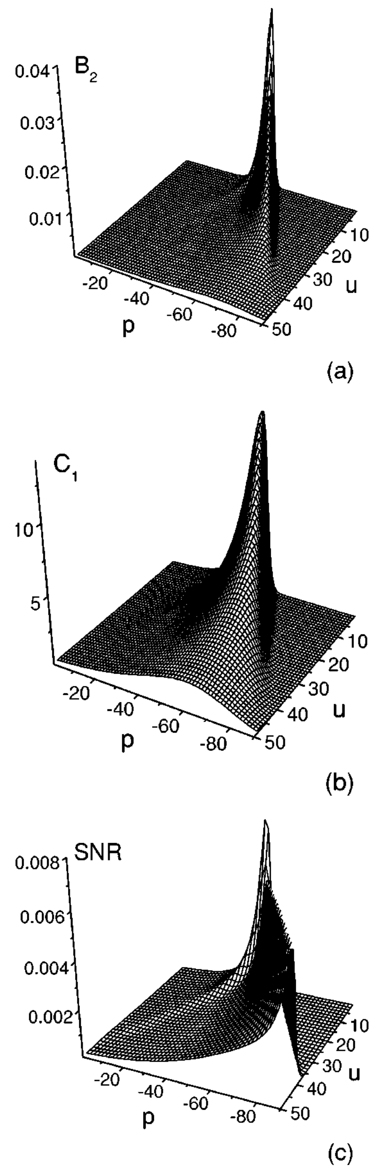

FIG. 6. The dependences of the output signal, the noise background, and the signal-to-noise ratio on the bias *p* and the noise amplitude *u* for the model (2) in the fast noise case [Eqs. (28) and (29)]. The parameter values are the same as in Fig. 3.

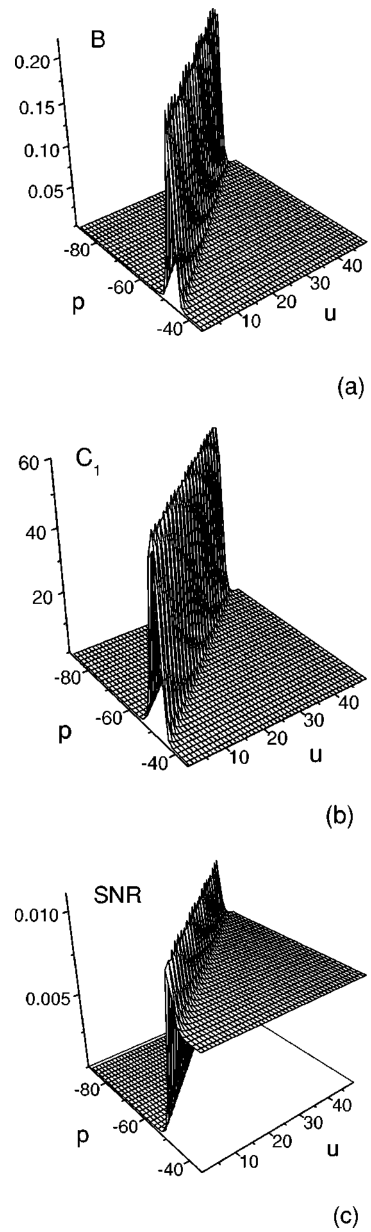

FIG. 7. The same graphs as in Fig. 6, but for the model (33).

universality of the described phenomenon, we use the simplified model of Hodgkin-Huxley type for transition probabilities, which is used widely in ion channel biophysics [15]:

$$\alpha_0(x) = a_2 \exp[b_2(x+p_2)],$$

$$\beta_0(x) = a_1. \qquad (33)$$

These curves intersect like the ones in Eq. (2) (Fig. 1), and one can expect that both systems behave similarly in the adiabatic range. Indeed, it is seen from Fig. 5 that for $|p|,u \gg 1$ all the quantities demonstrate the same behavior. Interestingly, stochastic resonance is observed for the model (33) only at $p < -p_2$. Such a universality disappears when the noise is fast, as it is demonstrated in Fig. 6 [the model(2)] and Fig. 7 [the model (33)]. In Fig. 6(b) we see that for the model (2) the noise background decreases with increase of *u*, demonstrating noise-induced reduction of noise. This interesting phenomenon was recently found in other simple model systems ([17,18]). The sections of the SNR surface are shown in Fig. 8 for the model (2). They look very similar to the curves of information gain for various biases presented in the paper of Goychuk and Hänggi [16]. Figure 7 demonstrates also that only one branch of resonance exists for the model (33).

Let us consider now the coherence function in the adiabatic case. We see from Eq. (25) that, since $B_1 \gg \gamma/2C$, $B_3 = 0$ for $u \gg 1$, and $v = 0$,

$$\Gamma^2(0) \approx 1, \quad u \gg 1. \qquad (34)$$

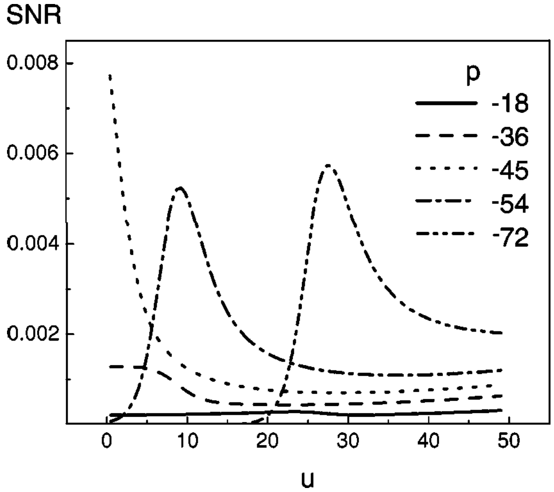

FIG. 8. The signal-to-noise ratio vs the noise amplitude u for the model (2) in the fast noise case for several values of the bias p. The curves are the sections of the surface depicted in Fig. 6.

This means that the output signal is completely synchronized with the external dichotomous noise. Figure 9 displays the graphs of coherence function for both studied models. When comparing these graphs with Figs. 3(b) and 5(b), we see that the synchronization takes place in the region where the noise background is large. Remember that for fast noise the signal and noise are desynchronized [Eq. (27)].

VI. FINAL REMARKS

In the present work we focus on the stochastic resonance arising in the range of parameters where the probabilities of transitions between two states of the channel are close to each other. When noise is strong enough, there are two branches of resonance, at the bias values above and below the threshold potential. In the adiabatic case, at values of parameters used by Goychuk and Hänggi in the model of the potassium channel [16], the values of SNR maximum in these branches are rather close; at the other relation of parameters they can strongly differ. Nevertheless, the very occurrence of such resonance is universal and requires only the presence of a point where the curves of dependences of transition probabilities on external potential intersect. Once again, we note that the kind of SR described here differs from that was found by Bezrukov and Vodyanoy ([11–14]), since the last is observed in the range $\alpha_0 \ll \gamma \ll \beta_0$ (Fig. 1). The theory developed by us is rather general, and one can investigate with it the other interesting and physically motivated ranges of parameters that remain beyond the scope of our paper.

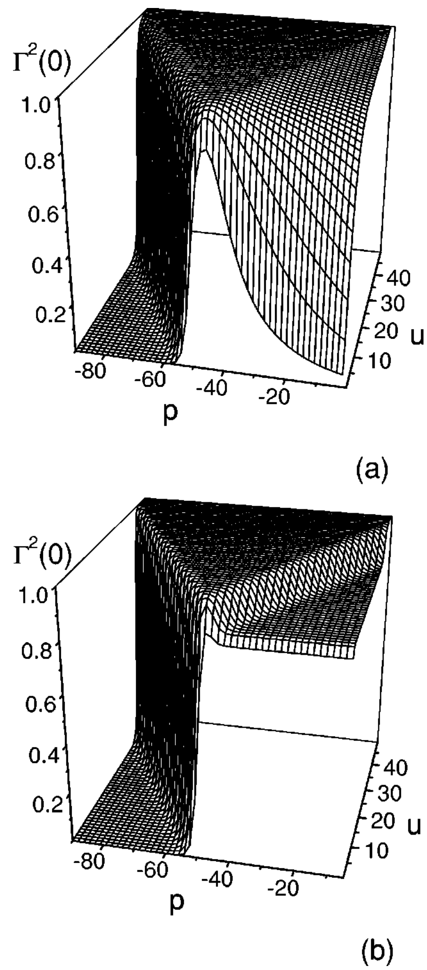

FIG. 9. The dependence of the coherence function maximum $\Gamma^2(0)$ in the adiabatic case [Eq. (25)] on the parameters p, u for the models (2) (a) and (33) (b) for the same parameter values as in Figs. 3 and 5.

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